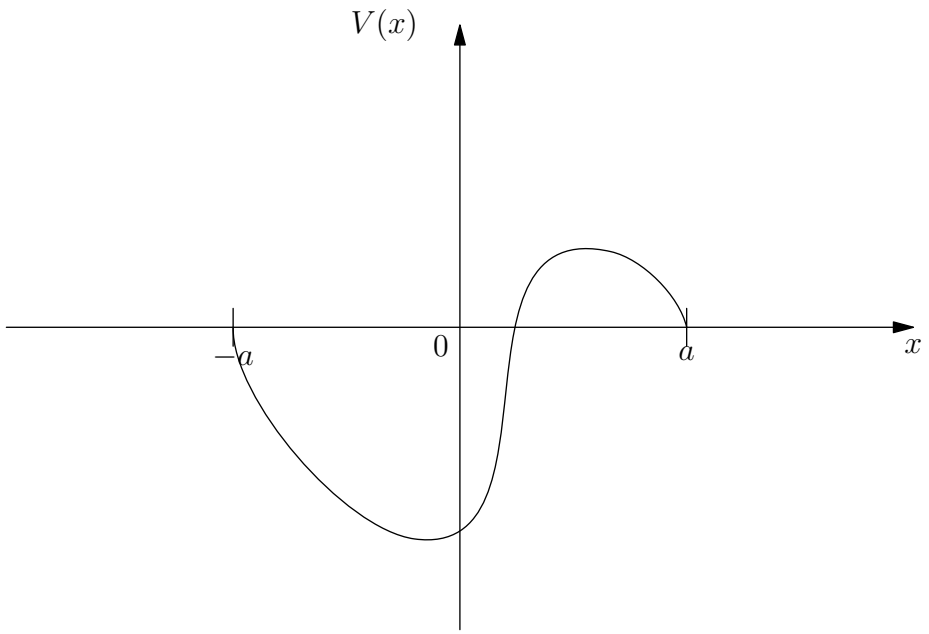
J04Q.1—Arbitrary Potential

Problem

Consider a particle in *one* dimension moving under the influence of a potential $\lambda V(x)$, with $V(x) \to 0$ as $|x| \to \infty$. The hamiltonian is

$$H = \frac{p^2}{2m} + \lambda V(x).$$

For simplicity, assume that the potential is non-zero only in the region $|x| < a$.



a) Use the variational principle to show that as long as $\int V(x)dx < 0$ this Hamiltonian has a bound state for *arbitrarily weak* but positive coupling constant λ.

b) Give an upper bound for the energy of this bound state for $\lambda \ll 1$.

c) Would the same approach work in three dimensions? Explain why.