J04T.1—Non-interacting Spins

Problem

Consider N non-interacting quantized spins in a magnetic field $\vec{B} = B\hat{z}$. The energy of the spins is $-BM_z$, where

$$M_z \equiv \mu \sum_{i=1}^{N} S_z^{(i)}$$

is the total magnetization. For each spin, $S_z^{(i)}$ takes only $2S + 1$ values $-S, -S + 1, \ldots, S - 1, S$. Given the temperature of the system T:

a)   Calculate the Gibbs partition function $Z(T, B)$;

b)   Calculate the Gibbs free energy $G(T, B)$ and evaluate its asymptotic behavior at weak ($\mu BS \ll k_B T$) and strong ($\mu B \gg k_B T$) magnetic field;

c)   Calculate the zero-field magnetic susceptibility

$$\chi \equiv \left(\frac{\partial M_z}{\partial B} \right)_{B=0}$$

d)   Calculate the magnetic susceptibility at strong fields $\mu B \gg k_B T$.