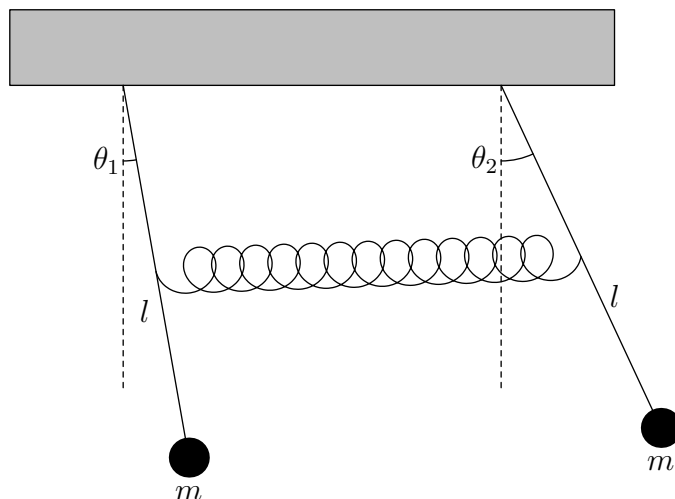


M04M.2—Coupled Pendula

Problem

Two simple pendula, each of length l and mass m are coupled by a spring of force constant k which is attached to their massless and inextensible rods at their halfway points. The spring is relaxed when the pendula are vertical.



- Write a Lagrangian for the system.
- Find the normal modes and their frequencies for small oscillations about equilibrium.
- At $t = 0$ the left pendulum is displaced by a small angle $\theta_1(0) = \theta_0$ and released from rest while the right pendulum is at rest with $\theta_2(0) = 0$. Find $\theta_{1,2}(t)$.
- How long will it be before the energy of the left pendulum is transferred completely to the right pendulum?