

## M04T.3—Binary Mixture of Atoms

### Problem

Consider a binary mixture of atoms labeled A and B. There are  $N \gg 1$  atoms of which  $xN$  are of type A and  $(1-x)N$  are of type B. The atoms occupy equidistant sites along a line. Ignoring the kinetic energy, the statistical mechanics is governed by the potential energy which is determined as follows: two neighboring A atoms or two neighboring B atoms contribute  $-\epsilon$  while a pair of neighboring ABs contribute  $-\epsilon/2$ . A sample arrangement may look like

... AAABABBBBAABAAA ...

- a) What is the average size of a cluster of A atoms at temperature  $T = \infty$  in the limit  $N \rightarrow \infty$ ?
- b) What is the average size of a cluster of A atoms at  $T = 0$  in the limit  $N \rightarrow \infty$ ?
- c) Calculate an estimate for the free energy of the mixture regarding the atoms to be independently and randomly distributed.
- d) Does the above value provide a variational bound on the free energy? (Upper, or lower bound?)
- e) Within this approximation, estimate the phase transition temperature at  $x = 1/2$ .
- f) Actually, this system has no phase transition at  $T = 0$ . Explain why.