J05Q.1 - Fermion Entanglement

Problem

Two spin 1/2 particles interact via the Hamiltonian

$$H = -J\vec{S}_A \cdot \vec{S}_B.$$

At time $t = 0$, spin A points in the positive z-direction and spin B points in the negative Z-direction. Compute the density matrix of spin A at time t. At which time does it describe a pure state, that is, at which time does the entanglement between the two spins vanish?