J05E.2 - Light Incident on a Medium

Problem

An electromagnetic wave of frequency ω propagates through vacuum along the z axis and is incident on homogeneous medium which fills space for $z \geq 0$. The medium has a magnetic permeability μ and real dielectric constant ϵ. The medium has a large conductivity σ that is a known real function of the frequency ω. Within the medium, the z dependence of the electric field amplitude is:

$$\vec{E} = \frac{1}{2}\{\vec{E}_0 e^{i(kz-\omega t)} + \vec{E}_0^* e^{-i(k^* z-\omega t)}\}$$

a) Obtain an expression for the complex propagation constant k in terms of ϵ, ω and σ.

b) Calculate the phase of the reflected wave, relative to that of the incident wave, at $z = 0$ in terms of ϵ, ω and σ.

c) A conductor with $\sigma = 10^{16}\, s^{-1}$ ($\sigma \simeq 10^6$ Ohm^{-1} m^{-1} in SI units) reflects 90% of the incident radiation. Assume the ϵ and μ are the corresponding values in vacuum. What is the frequency ω of the incident wave?