J05T.3 - Thermodynamic Variables

Problem

A thermodynamic system has the following relation between its entropy S, volume V, internal energy U, and particle number N:

$$S(U, V, N) = \gamma (UVN)^{1/3},$$

where γ is a constant.

- a) Derive a relation connecting U, N, V and the temperature T.
- b) Find the heat capacity C_{VN} at constant V and N, as a function of V, N, and temperature T.

Now assume you are given two identical bodies with the above properties. N and V are the same for both, and are fixed, but the two bodies have different initial temperatures, T_1 and T_2 .

- c) If the two bodies are placed in thermal contact, and left alone until heat flow ceases and equilibrium is reached, what is their common final temperature T_f ?
- d) If the flow of heat between the bodies is used to drive an engine that does the maximum possible amount of useful work W_{max} before the two bodies reach a common final temperature T_f' , what is that temperature? What is W_{max} ?