M05M.1 - Vibration Damping With a Piston

Problem

Sometimes it is required to reduce the influence of vertical floor vibration on experimental setups. To accomplish this, the experimental apparatus (of mass M) is mounted on gas-filled pistons (which have face area A and equilibrium volume V_0). The gas pressure in the cylinder is much larger than atmospheric pressure. The pistons are typically equipped with a mechanism to dampen their oscillations (by immersing part of the piston in oil), so that there is a retarding force, $-vM/\tau$, where v is the velocity of the piston and τ is the characteristic damping time.

a) Ignoring damping ($\tau \to \infty$), determine the resonance frequency ω_0 of the system for small vibrations. Explain whether any changes in state of the gas are more nearly isothermal or adiabatic.

b) Characterize the performance of this vibration-isolation system as a function of frequency by calculating its transmission coefficient, i.e. the ratio of the vibration amplitude of the apparatus to a small vibration in the floor. Include any phase shifts and express your answer in terms of ω_0 and the damping time τ. For which frequency range do you expect this system to be useful for isolating the vibrations of the floor?

c) How should the system parameters be chosen to provide as much vibration isolation as possible, subject to the constraint that a sudden displacement of the floor causes no oscillations in the system?