M05E.3 - Electron Orbit Decay

Problem

In the Bohr model of the hydrogen atom's ground state, the electron moves in a circular orbit of radius $a_0 = 0.53 \times 10^{-10}$ m around the proton, which is assumed to be rigidly fixed in space. Since the electron is accelerating, a classical analysis suggests that it will continuously radiate energy, and therefore the radius of the orbit would shrink with time.

a) Assuming that the electron is always in a nearly circular orbit and that the rate of radiation of energy is sufficiently well approximated by classical, nonrelativistic electrodynamics, how long is the *fall time* of the electron, i.e. the time for the electron to spiral into the origin?

b) The charge distribution of a proton has a radius of about 10^{-15} m, so the classical calculation would be modified once the radius of the electron's orbit is smaller than this. But even before this, modifications may be required due to relativistic effects. Based on the analysis of part a), at what radius of the electron's orbit would its velocity be, say $0.1c$, where c is the speed of light, such that relativistic corrections become significant? What fraction of the electron's fall time remains according to part a) when the velocity of the electron reaches $0.1c$?

c) Do the relativistic corrections increase or decrease the fall time of the electron?