M05M.3 - Satellite Orientation

Problem

The orientation of a satellite in low-Earth orbit can be stabilized with gravity gradients. Let the satellite be a long cylinder of length L and radius a ($a \ll L$). The distribution of the mass m is uniform within the cylinder. The satellite is in a circular orbit around Earth with a period T_{or} . Express your answers to parts b), c) and d) in terms of T_{or} , L and a.



- a) What is the stable equilibrium orientation, for which the satellite appears to be at rest in a coordinate system rotating with the same angular velocity as the orbital motion?
- b) What is the period T_{π} of small oscillations about equilibrium in the orbital plane?
- c) What is the period T_{σ} of small oscillations about equilibrium perpendicular to the orbital plane?
- d) The satellite is given a small angular speed ω around its long axis. To first order in ω , find the new stableequilibrium orientation of the satellite, where the direction of the long axis appears to be fixed in the rotating coordinate system.