M05T.3 - Surface Waves and Heat Capacity (M04T.2)

Problem

Consider waves on a liquid surface where the restoring force is produced by the surface tension. Assume there is a single polarization and the dispersion relation is

$$\omega^2 = \frac{\gamma}{\rho} k^3,$$

where γ is the surface tension of the liquid, ρ is its density, ω is the frequency of the waves and k is their wavenumber. Our goal is to find the contribution of these waves to the low temperature heat capacity of the liquid. Surface wave excitations are quantized (quasiparticles, like phonons!).

a) If the surface is in equilibrium at temperature T, what is the average energy of a wave with frequency ω? (Ignore the $\hbar\omega/2$ zero point energy.)

b) At low temperatures what are the energy per unit area and heat capacity per unit area of these surface waves?