J06E.1 - Dielectric Cylinder in an Electric Field

Problem

An infinitely long cylinder of radius a and dielectric constant ϵ is placed in an initially uniform electric field of strength E_0. The axis of the cylinder is oriented at a right angle to the direction of the field.

a) Find the electric potential $\Phi(r, \theta, z)$ both inside and outside of the cylinder, in cylindrical coordinates (r, θ, z), where the z axis is the axis of the cylinder.

b) Find the electric fields $\boldsymbol{E}$ and $\boldsymbol{D}$ inside the cylinder.

c) What is the surface polarization (bound charge) density σ_b at $r = a$? What is the volume polarization charge density ρ_b for $r < a$?

d) What is the electrostatic energy per unit length inside the cylinder?