J06T.1 - Bose Einstein Condensation

Problem

A spin-zero particle of mass m moves nonrelativistically in the 3-D harmonic potential given by

$$V(x, y, z) = \frac{m\omega^2}{2}(x^2 + y^2 + z^2).$$

a) Obtain an expression for $D(\epsilon)$, the density of states for this particle, that is valid at energies much larger than $\hbar\omega$, where the energy ϵ can be approximated as a continuous variable.

b) Suppose there are now N (where N is large) noninteracting spin-zero particles in this harmonic oscillator potential. The particles are in equilibrium at temperature T, with $k_B T \ll \hbar\omega$. What is the chemical potential of the system in this low T regime (including the leading dependence on N for large N)?

c) In the thermodynamic limit of large N, this system has a Bose-Einstein condensation (BEC) such that the number of particles in the ground state is large even for temperatures well above $\hbar\omega$. The number of particles in the ground state is $N_0(T) = N(1 - (T/T_E)^\alpha)$, where T_E is the Einstein condensation temperature. Determine the exponent α and an expression for T_E. You may encounter a dimensionless integral whose value is not readily evaluated; you may give your answers in terms of this integral. For T_E to remain finite in the thermodynamic limit, the trap must be "softened", so that ω decreases in the appropriate way as N grows in order to keep T_E finite in the limit.