J06Q.2 - Two Indistinguishable Bosons

Problem

Consider two indistinguishable nonrelativistic **bosons** of mass m, constrained to move one dimensionally around a circle of perimeter L (equivalently, on a line of length L with periodic boundary conditions). The two particles interact via a potential that is a delta-function, $V(x_1, x_2) = g\delta(x_1 - x_2)$. This interaction may be of either sign and of any strength. Give answers for all values of g, including g = 0.

- a) The particles have spin zero. Solve for the wave function, energy and degeneracy of the ground state(s). Some of your answers here and below may involve a parameter that you may define as the solution to a transcendental equation.
- b) Now the particles have spin one. The interaction is spin-independent. Again, find the wave function(s) (including the spin component), energy, degeneracy and total spin of the ground state(s). Also, find the total spin and the degeneracy of the lowest-energy excited state(s) for each value of g.