# J06M.3 - Bead on a Rotating Wire (J04M.1)

## Problem

A bead of mass $m$ slides without friction on a wire. At time $t = 0$ the wire is in the $x$-$z$ plane and has shape

$$z = a\left(\frac{|x|}{a}\right)^{\alpha},$$

with $a > 0$ and $\alpha > 0$. The wire is rotating about the $z$ axis with a constant, nonzero angular velocity $\omega$, without changing this shape. Earth's gravity causes a force $mg$ on the bead along the $= \hat{z}$ direction.

a) Find the equation of motion of the bead.

b) For each value of $a > 0$ and $\alpha > 0$, find all solutions to this equation of motion where the bead is not moving with respect to the wire.

c) For each solution you found in part b), determine whether there are nearby solutions with the bead undergoing small amplitude *harmonic* motion with respect to the wire, and solve for the frequency of those oscillations when they exist.

d) For what values of $a > 0$ and $\alpha > 0$ are there solutions where the bead starts near the $z$ axis with a finite speed, but then "escapes" to infinitely far from the axis? When this happens, what is the trajectory of the bead at long times (assuming the dynamics remains nonrelativistic)?