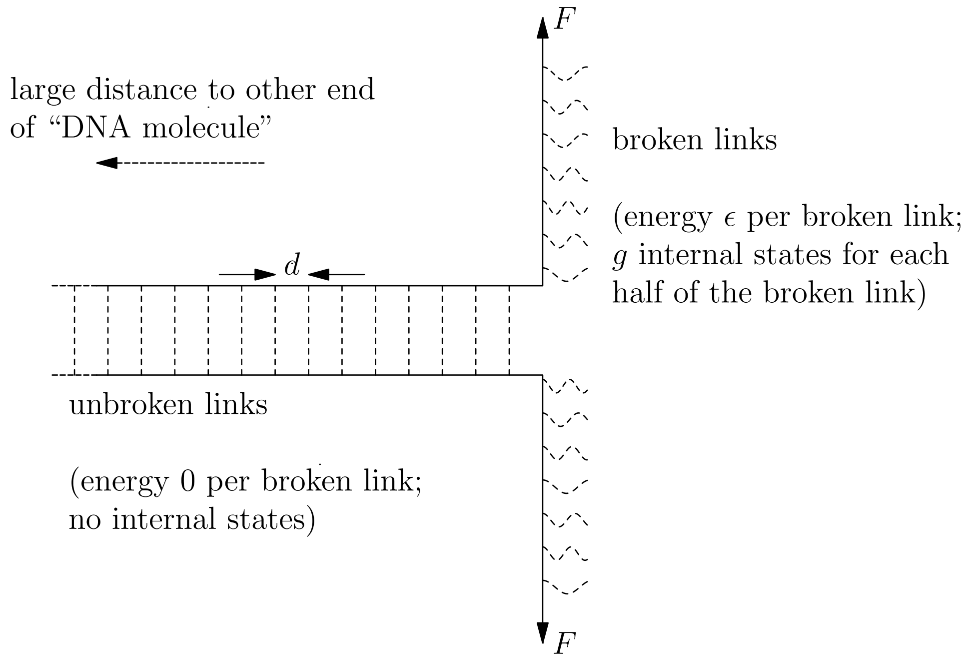
J06T.3 - DNA Molecule

Problem

A simple "toy model" model for how complementary strands of DNA are bound together resembles a zipper (see figure). The two strands are connected by "links" (base pairs) spaced at equal intervals d along the strands. It costs an energy ϵ to break a link, and a link can only be broken if its neighbor to the right is also broken. An unbroken link is a unique internal state, but each of the two dangling ends of a broken link can be one of g internal states.



At the right-hand end of the DNA molecule, the experimenter applies a tension force F to each of the two strands to try to separate them. This force is not strong enough to separate the chains at $T = 0$.

a) Assume that $g = 1$ (so broken links have no internal states). At finite temperatures $k_B T \gg \epsilon$, what is the mean number $\bar{n}$ of broken links near the ed of the DNA molecule, when $F = 0$? (Assume that $\bar{n}d$ is much smaller than the total length L of the DNA molecule.) How does it change when the force is applied?

b) Now assume that $g > 1$. Write down the configurational partition function, and obtain the free energy associated with the links between the strands. Obtain the critical temperature $T_c(g, \epsilon, F, d)$ above which the two strands of an infinitely long DNA molecule would be pulled apart by the applied force F.

c) Obtain an expression for $\bar{n}(T, g, \epsilon, F, d)$ valid for an infinitely-long DNA molecule at all temperatures less than T_c (including $k_B T \ll \epsilon$), and make a sketch showing its principal features.