M06T.1 - Interacting Particles on a Line

Problem

Consdier a system of N classical particles on a line with Hamiltonian

$$H = \frac{p_1^2}{2m} + U_1(x_1) + \sum_{i=2}^N \frac{p_i^2}{2m} + U(x_i - x_{i-1}).$$

The potential between neighboring particles is of the form:

$$U(y) = \begin{cases} +\infty, & \text{if } y < 0; \\ -U_0, & \text{if } 0 \le y \le a; \\ 0, & \text{if } a < y. \end{cases}$$

Here both U_0 and a are positive. A constant force f is applied to the rightmost particle i = N.

- a) Compute the mean length, $\langle x_N \rangle$, of the system as a function of N, T, and f.
- b) Obtain the high and low temperature limits of the result from part a).