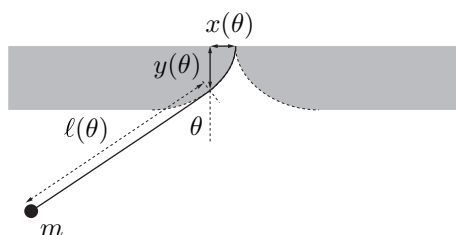


M06M.2 - Huygens' Pendulum (M92M.1)

Problem

To compensate for the fact that the period of a simple pendulum depends on the amplitude of oscillation, the 17th century Dutch physicist Christian Huygens devised the following setup, depicted in the figure below. It shows a simple pendulum consisting of a mass m and a string of length ℓ_0 whose motion is constrained by a cusp shaped piece of wood. The problem is to determine the shape of the wooden surface so that the period of the pendulum is independent of the amplitude.



- Parametrize the shape of the surface by $x(\theta)$ and $y(\theta)$, as indicated in the figure. Write the Lagrangian for the pendulum.
- What property must the Lagrangian have in order for the period of oscillation to be independent of the amplitude? Find the required shape $(x(\theta), y(\theta))$.