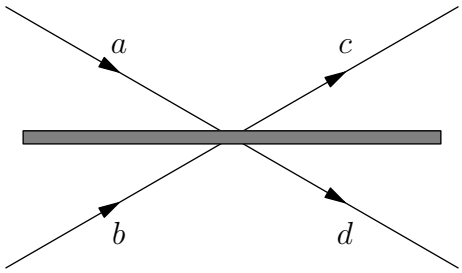
M06Q.2 - EPR Beam Splitter

Problem

In this problem, we consider two-photon interference at a beam splitter. The aim is to that, with a suitable measurement, the two photons can be projected onto an EPR-pair state after leaving the beam splitter. The beam splitter has two spatial input modes $|a\rangle$ and $|b\rangle$, corresponding to the two sides of a semi-transparent mirror:



Initially, photon 1 is horizontally polarized and arrives along the direction $|a\rangle$,

$$|\psi_i\rangle_1 = |\leftrightarrow\rangle_1 |a\rangle_1$$

while photon 2 is vertically polarized and arrives along the other direction $|b\rangle$:

$$|\phi_i\rangle_2 = |\updownarrow\rangle_2 |b\rangle_2.$$

The beam splitter is a semi-reflecting mirror, and maps the two spatial input states to a linear sum of a reflected and a transmitted state:

$$|a\rangle \rightarrow \frac{1}{\sqrt{2}}(i|c\rangle + |d\rangle)$$
$$|b\rangle \rightarrow \frac{1}{\sqrt{2}}(|c\rangle + i|d\rangle)$$

The factor i corresponds to a phase jump upon reflection at the semi-transparent mirror.

a) Write the output state $|\psi_f\rangle_i$ of photon i after it leaves the beam splitter.

b) Now write the total output state of the two photons. Remember: photons are bosons!

A measurement shows that the two photons leave on opposite sides from the beam splitter.

c) Give the probability that a position measurement of the photons yields this outcome.

d) What is the polarization state of the two photons after this measurement?

e) Write the density matrix that describes the polarization state of one of the photons.