M06T.2 - Diatomic Gas in an Electric Field (J07T.2)

Problem

The Hamiltonian for a diatomic molecule with constant dipole moment μ in a homogeneous electric field $\vec{E} \equiv (0, 0, E)$ is:

$$H = \frac{1}{2M}(p_x^2 + p_y^2 + p_z^2) + \frac{1}{2I}p_\theta^2 + \frac{1}{2I\sin^2\theta}p_\phi^2 - \mu E\cos\theta$$

(M = mass of molecule, I = moment of inertia, and (r, θ, ϕ) are polar coordinates). Consider an ideal gas of N such *classical* molecules in a volume V, using *Boltzmann statistics*.

a) Compute the free energy $F_N(T, V, E)$.

b) Compute the dipole moment per unit volume ("polarization"), $\boldsymbol{P}_N(T, V, E)$, of the gas and evaluate the dielectric constant ϵ in the limit $\mu E \ll k_B T$.

[Recall: $\epsilon \boldsymbol{E} = \boldsymbol{E} + 4\pi \boldsymbol{P}$.]