M06Q.3 - Two Interacting Particles

Problem

Consider two particles of mass m moving in one dimension. Particle 1 moves freely, while particle 2 experiences a harmonic potential $V(x_2) = \frac{1}{2}m\omega^2 x_2^2$. The two particles interact via a delta function potential

$$V_{int}(x_{12}) = \lambda\delta(x_{12}),$$

with $x_{12} \equiv x_1 - x_2$. Particle 2 starts in the ground state $|\psi_0\rangle$, and particle 1 comes in from the left in a momentum eigenstate $|p_i\rangle$. Compute the transition probability $\mathcal{P}_{01}$ that particle 2 ends up in the first excited state $|\psi_1\rangle$, to leading order for small λ.