J07E.1 - Point Charge and Conducting Sphere

Problem

A point charge Q is located at a distance r away from the center of a thin spherical *conducting* shell of radius a, which has a net charge also equal to Q. Let $U(r)$ be the total electrostatic potential energy of this system.

a) What is $U(0) - U(\infty)$?

b) Determine the leading behavior of $U(r) - U(\infty)$ as $r \to a$, and make a qualitatively correct sketch showing its important features over the whole range $0 \leq r < \infty$.

c) As $r \to \infty, U(r) - U(\infty) \to Q^2/4\pi\epsilon_0 r$. Obtain the leading correction to this behavior for large r.

d) If you have not already done so, give the explicit function $U(r) - U(\infty)$ for all r.