J07T.1 - Dilute Ionized Hydrogen

Problem

Consider an extremely dilute gas of partially ionized atomic hydrogen, such as occurred in the early universe. The binding energy of an electron and proton in the atomic ground state is $\epsilon \simeq 13.6$ eV. Let this dilute plasma be neutral, with equal numbers of electrons and protons. Assume it is at equilibrium at a temperature T such that $\epsilon/(k_B T) = 100$.

a) The fraction of the atoms ionized is $1/2$, so the densities of atoms, free electrons, and free protons are all equal. What is this density? Give it as a formula in terms of T, ϵ, and any fundamental constants or particle masses, and then, for $\epsilon/k_B T = 100$, give a numerical density, in units of m-3, correct to the nearest order of magnitude Make and justify any appropriate approximations that will simplify your calculation.

b) At this density, estimate how much you need to lower the temperature to reduce the fraction of ionized atoms to $1/10$, so only 10 % of the electrons are free, while the remainder are bound in atoms. Again, make and justify any reasonable approximations

Useful constants: $\hbar c = 2000$ eV-Å $= 200$ eV-nm; $m_e c^2 \simeq 500,000$ eV.