

## J07E.2 - Rotating Shell of Charge

### Problem

A hollow spherical shell centered at the origin has radius  $a$  and a total electric charge  $Q > 0$  uniformly distributed over its surface. The shell is slowly spun up to an angular velocity  $\omega = \omega_0 \hat{z}$  (where  $\omega_0 > 0$ ) over a period of time  $\tau \gg a/c$ , where  $c$  is the speed of light, so radiation effects can be ignored.

- a) To linear order in  $d\omega/dt$ , find expressions for the electromagnetic fields  $\vec{E}(\vec{r})$  and  $\vec{B}(\vec{r})$  throughout space, as functions of  $\omega$  and  $d\omega/dt$ . Make a qualitatively correct sketch showing the pattern of electric field lines in the plane  $z = 0$ . Indicate the direction of rotation of the charged shell on your plot.
- b) After the angular velocity  $\omega_0$  is reached, what is the total angular momentum  $\vec{L}$  stored in the electromagnetic fields?