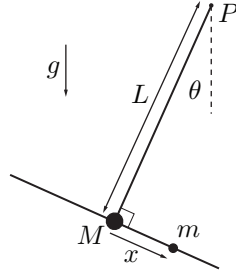# J07M.2 - Inverted T Pendulum

Problem

Two massless rods are rigidly joined at right angles in a rigid inverted "T" shape, with a point mass M attached to the junction. A second point mass m is free to slide frictionlessly along the first rod that forms the base of the inverted "T", as shown. (Its motion is not impeded by the mass M.)

This whole arrangement is suspended in Earth's gravity from a frictionless pivot P attached to the end of the second rod (see figure), where the distance from the pivot to the junction of the rods is L. Let θ be the angle of the second rod relative to the vertical, and x be the displacement of the mobile mass m relative to the fixed mass M, as shown.

a)   Initially, the system is at rest with $\theta = 0$ and $x = 0$. Analyze the linearized equations of motion about this stationary state for small values of θ and x, to find the eigenmodes. For each eigenmode, determine whether it is stable or unstable.

b)   At time $t = 0$ an instantaneous small horizontal impulse J is applied to the mass m, so that immediately afterwards $(t = 0^+)$ it has finite velocity along the horizontal rod $(\dot{x} > 0)$, but $\dot{\theta} = 0$. Solve the linearized equations of motion for the subsequent motion $x(t)$ and $\theta(t)$.