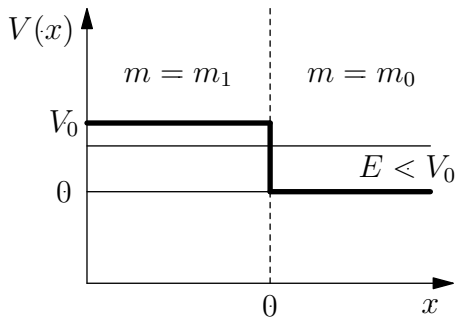
J07Q.2 - Effective Mass

Problem

An electron is moving in one dimension in a potential $V(x) = 0$ for $x > 0$ and $V(x) = V_0 > 0$ for $x < 0$. The region $x > 0$ is empty space, where the electron mass is the usual bare mass m_0, but in the region $x < 0$ it has a modified "effective mass" m_1. When the mass of a non-relativistic particele depends on its position, the Hamiltonian should be written in the operator-ordered form

$$H = \frac{1}{2}p(m(x))^{-1}p + V(x)$$

where $[x, p] = i\hbar$.



a) The standard continuity conditions (continuity of $\Psi(x)$ and $\Psi'(x) \equiv d\Psi(x)/dx$) only apply at $x = 0$ if $m_1 = m_0$. Derive the continuity conditions that apply at points where the mass is discontinuous.

b) The (unnormalized) wave function of an eigenstate of the Hamiltonian with an energy $E < V_0$ is given by $\Psi(x) = A \sin k(x - x_0)$ for $x > 0$. Find k, x_0 and $\Psi(x)$ for $x < 0$. Make a sketch of the function $\Psi(x)$, indicating its essential features.