J07E.3 - Negative Dielectric Constant

Problem

Electromagnetic waves with (angular frequency ω can propagate without loss in a linear isotropic medium where the (frequency-dependent) dielectric constant $\epsilon(\omega)$ and relative magnetic permeability $\mu(\omega)$ are both real, and their product is positive. If ϵ and μ have no frequency dependence, electromagnetic stability requires $\epsilon > 0$ and $\mu > 0$, but if they are frequency-dependent, this condition becomes $(\omega\epsilon)' \equiv d(\omega\epsilon)/d\omega > 0$ and $(\omega\mu)' \equiv d(\omega\mu)/d\omega > 0$. This allows $\epsilon(\omega)$ and $\mu(\omega)$ to be negative at some frequencies, if they have strong frequency dependence.

While strong frequency dependence of ϵ is common, most materials have $\mu \simeq 1$ at optical frequencies. Recently, however, artificial structures where ϵ and μ are *both* negative in some frequency range have been created.

a) Solve the Maxwell equations for an electromagnetic wave in a unifrom medium that is isotropic and time-reversal invariant: $\epsilon(\omega)$ and $\mu(\omega)$ are real, even functions of ω in the frequency range of interest, and c is the speed of light in vacuum. For $\vec{X} = \vec{E}, \vec{D}, \vec{B}$ and $\vec{H}$, find the electromagnetic fields $\vec{X}(\vec{r}, t) = \vec{X}_0 \cos(\vec{k} \cdot \vec{r} - \omega t)$. Give the wavenumber $|\vec{k}| = k(\omega)$ as a function of (angular) frequency ω.

b) Obtain an expression for the ratio $(\omega/k)/v_g$, where $\omega/k(\omega)$ is the *phase velocity*, and $v_g(\omega)$ is the *group velocity*, the velocity at which energy propagates. Show that $v_g(\omega)$ is negative (and *antiparallel* to the phase velocity) if $\epsilon(\omega)$ and $\mu(\omega)$ are negative.

c) A narrow beam of light from a laser operating at frequency ω is incident on the flat surface of an isotropic medium with negative $\epsilon(\omega)$ and $\mu(\omega)$ (and hence negative $v_g(\omega)$) at an angle θ_i relative to the normal. The wavenumber of waves with frequency ω in the medium is $k(\omega)$. What is the angle θ_r made by the refracted beam in the medium? Make a diagram showing a possible path of the incident and refracted beams. Assume that the incoming beam is traveling in a vacuum. (You may find it helpful to consider the relation between the incident and refracted photon wavevectors $\vec{k}_i$ and $\vec{k}_r$.)