# J07M.3 - Bubble in an Incompressible Fluid

## Problem

An explosion at time $t = 0$ in an ideal (zero viscosity) incompressible fluid produces a perfectly spherically symmetric expanding bubble of vacuum with radius $R(t)$ (neglect the effect of any gas or vapor inside the bubble). The bubble expands to maximum radius $R_{max}$ and then collapses. The pressure in the fluid far from the bubble is $P_\infty$, and the mass density of the fluid is $\rho$. Neglect any effects of surface tension or gravity; assume the bubble remains spherically symmetric at all times, and that the velocity field in the fluid is purely radial.

a) Obtain an expression for the velocity field inside the fluid, and hence get an expression for the total energy (kinetic + potential) of the fluid in terms of $R$ and $dR/dt$.

b) Obtain an equation of motion for the bubble's radius $R(t)$ of the form

$$\frac{dR}{dt} = f(R)$$

What is the function $f(R)$?

c) How long does it take for the bubble to collapse after it reaches its maximum radius? Your answer can contain a finite dimensionless integral whose value you have not obtained.

d) What is the asymptotic behavior of $R(t)$ in the final moments of the bubble's collapse when $R \ll R_{max}$? (Do **not** consider the possibility of "cold fusion"!)