J07T.3 - Spin Waves

Problem

Consider spin waves in an isotropic ferromagnetically ordered crystal. These are waves in which the spins on each atom oscillate in space and time. Just as with sound waves, the spin waves can be quantized and they can store internal energy in a crystal lattice. However, these waves have a different relation between frequency and wavenumber than do sound waves. In particular, at low wavenumber.

$$\omega(k) = Ak^2$$

where A is a constant. Consider a crystal containing N spins in thermal equilibrium at temperature T.

a) What is the average energy in a spin wave mode of frequency ω? (Neglect the zero-point energy of the mode).

b) At low temperatures, the heat capacity of the spin wave system in the crystal is proportional to T^α. What is the numerical value of α?

c) If the material is a metal, do the spin waves give the dominant contribution to the heat capacity in the low-temperatue limit? What if the material is an insulator? Explain both of your answers.