M07T.1 - Pions

Problem

a) The π meson is a spinless boson which can exist in three states of charge: π^+, π^0, and π^-. We assume its mass to be zero, which is a good approximation at high enough temperatures. We therefore are in the ultrarelativistic case where the dispersion relation (in units where $c = 1$) is given by $\varepsilon_p = |\vec{p}| = p$. Calculate (in units where $\hbar = 1$ and $k_B = 1$) the pressure $P_\pi(T)$ of a gas of π mesons at temperature T and chemical potential 0.

[You may need $\int_0^\infty dx\, x^3(e^x - 1)^{-1} = \pi^4/15$.]

b) The π mesons are made of "up" and "down" quarks and antiquarks, which are spin-1/2 fermions that can be considered massless, interacting via the exchange of "gluons", which are massless spin one bosons. At low temperatures the quarks and gluons don't exist as free particles, they are confined inside the mesons. As the temperature rises, a transition takes place that frees the quarks and gluons. We thus obtain a gas, assumed ideal, of quarks and gluons: the *quark-gluon* plasma. Taking into account "colour" charges, this gas has a total of 24 fermion (quark) degrees of freedom and 16 boson (gluon) degrees of freedom. Calculate the pressure of this ideal gas $P_{plasma}(T)$ for zero quark and zero gluon chemical potential.

[You may need $\int_0^\infty dx\, x^3(e^x + 1)^{-1} = 7\pi^4/120$.]

c) The above expression should be modified in order to take into account the effect of interactions: if the plasma is in a box of volume V one has to add to its free energy a volume contribution BV, where B is a positive constant. How is the expression for P_{plasma} modified?

d) Compare the expressions for the pressure of the π meson gas and the quark-gluon plasma. Show that at low temperature the meson phase is stable, while at high temperature the plasma phase is stable. Find the phase transition temperature T_c.