M07Q.2 - Scattering

Problem

Consider elastic scattering from a 3-dimensional spherically symmetric square potential well of radius R:

$$V(\boldsymbol{r}) = \begin{cases} -V_0, & |\boldsymbol{r}| < R \\ 0, & |\boldsymbol{r}| \geq R \end{cases}$$

A beam of particles with mass m is impinging on the potential well with incident momentum p_0 parallel to the $\hat{z}$ axis.

a) Calculate the differential scattering cross-section as a function of the scattering angle θ in the Born approximation for $p_0 \ll \hbar/R$.

b) Sketch the differential cross-section as a function of the scattering angle θ and describe any prominent features.

c) Suppose now we have an infinite two-dimensional array of square wells in the x-y plane separated by distance $a \gg 2R$:

$$V_a(\boldsymbol{r}) = \sum_{j,k=1}^{\infty} V(\boldsymbol{r} - ja\hat{x} - ka\hat{y}).$$

Describe qualitatively the scattering pattern in this case and sketch the behavior of the cross-section, labeling the position of prominent features.