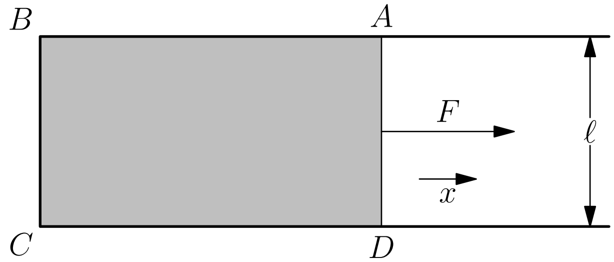
M07T.2 - Soap Film

Problem

A soap film is held by the four sides of a rectangle $ABCD$. The wire AD, of length ℓ, can be moved while kept parallel to BC allowing the film to be stretched. The strength of the force $\vec{F}$, applied in the positive x direction (*i.e.*, in the direction parallel to BA and CD), needed to keep AD fixed is $\sigma\ell$, where σ is the surface tension.



a) Denoting by $U(T, x)$ the internal energy of the soap film, where T is the temperature and $x = |AB|$, write the equation for TdS, where S is the entropy.

Now define the specific heat at constant length as

$$C_x = T\,\frac{\partial S}{\partial T}\bigg|_x\,.$$

For a wide range of temperatures close to room temperature, C_x is approximately constant (both with respect to T and x) and the surface tension varies linearly with T:

$$\sigma = \sigma_0(1 - a(T - T_0))$$

where σ_0, a and T_0 are positive constants.

b) We stretch the film by dx *quasi-statically* and at *constant temperature*. Calculate the corresponding infinitesimal increase of internal energy dU. Verify thet heat energy must be given to the film on order to maintain its constant temperature.

c) We now stretch the fillm *quasi-statically* and *adiabatically* by dx. Calculate the resulting temperature variation dT that accompanies this stretching. Is it positive or negative?

d) Sketch a reversible Carnot cycle (*i.e.*, a reversible cycle built out of two isothermal and two adiabatic transformations) on a σ-x diagram and indicate the x dependence of the adiabatic and isothermal curves.