M07M.3 - Mass on a Massive String

Problem

A point mass M is freely hanging from a string of mass m and length l in the presence of gravity. The upper end of the string is held fixed. You can assume that $M \gg m$ so the tension in the string is approximately constant.

- a) Write down the wave equation for small transverse oscillations of the string and the boundary conditions at the end of the string.
- b) Determine the transcendental equation whose solutions give the oscillation frequencies of the normal modes on the string.
- c) Approximately find the first two solutions of the transcendental equation for $M \gg m$ and give their physical interpretation.