# J08E.1 - Radiation from an Antenna

## Problem

An antenna consists of a circular wire loop of radius $R$, centered in the $x$-$y$ plane of a Cartesian coordinate system. The current has the same amplitude, $I = I(t)$, at all locations in the wire at a given time $t$. There is no net electrical charge on the wire. Assuming that $\dot{I}$, the rate of change of the current, is slow enough that magnetic dipole radiation dominates any higher multipoles, calculate:

a) the vector potential $\vec{A} = \vec{A}(\vec{r}, t)$ and scalar potential $\Phi$ at the location $\vec{r}$ and time $t$ when $r \gg cI/\dot{I}$ (specify your choice of gauge);

b) the magnetic and electric fields, $\vec{B}$ and $\vec{E}$, at $\vec{r}$ and $t$;

c) the energy flux, $S = S(\theta, \phi)$, as a function of the polar angles $\theta$ and $\phi$;

d) the total radiated power $P = \int S \sin\theta \, d\theta \, d\phi$.

Retain enough terms of any expansion in powers of $1/r$ to account for radiation. Insofar as possible, express your answers in terms of the magnetic dipole moment, $m = \pi R^2 I/c$, and its time derivatives.