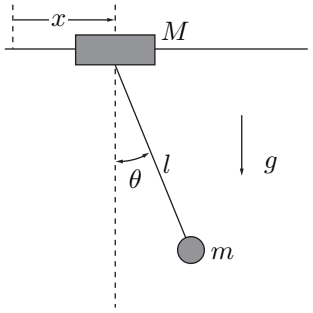
J08M.1 - Pendulum on a Sled

Problem

A plane pendulum consists of a bob of mass m suspended by a massless rigid rod of length l that is hinged to a sled of mass M. The sled slides without friction on a horizontal rail. Gravity acts with the ususal downward acceleration g.



a) Taking x and θ as generalized coordinates, write the Lagrangian for the system.

b) Derive the equations of motion for the system.

c) Find the frequency ω for *small* oscillations of the bob about the vertical.

d) At time $t = 0$ the bob and the sled, which had previously been at rest, are set in motion by a sharp tap delivered to the bob. The tap imparts a horizontal impulse $\Delta P = F\Delta t$ to the bob. Find expressions for the values of $\dot{\theta}$ and $\dot{x}$ just after the impulse.