J08Q.3 - Photoelectric Effect

Problem

Compute the differential cross section for the photo-electric effect, i.e., the scattering process by which a photon is absorbed by an atom while kicking an electron out of its orbit. Assume that initially the electron is in the ground state $|\psi_{100}\rangle$ of an H-atom,

$$\psi_{100}(\vec{r}) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$$

where a_0 denotes the Bohr radius. The incoming photon beam consists of N photons, all in a momentum and polarization eigenstate $|\vec{k}, \hat{\epsilon}\rangle$. The beam and atom are inside a periodic box with volume V. The final state has $N-1$ photons, and you may assume that the electron ends up in a momentum eigenstate $|\vec{k}_f\rangle$.

Hint: use the dipole approximation, where the interaction describing the coupling between the photon field and the electron is given by $(e/m)\vec{A} \cdot \vec{p}$, with

$$\vec{A} = \sqrt{\frac{2\pi\hbar}{V}} \sum_{\vec{k},\hat{\epsilon}} \frac{1}{\sqrt{c|\vec{k}|}} (a_{\vec{k},\hat{\epsilon}} + a^\dagger_{\vec{k},\hat{\epsilon}}) \hat{\epsilon}\,.$$

Here, $a_{\vec{k},\hat{\epsilon}}$ and $a^\dagger_{\vec{k},\hat{\epsilon}}$ are the photon creation and annihilation operators, $\hbar\vec{k}$ is the momentum and $\hat{\epsilon}$ the polarization of a photon.