J08T.3 - Motion of a Bead

Problem

A spherical particle with radius $r = 1\ \mu$m is placed in water at room temperature ($T = 298$ K). The density of the particle is close to that of the water (1 g/cm^3), and the (dynamic) viscosity of water is $\eta = 0.01$ poise $= 0.01$ g/(cm·s).

a) If the particle has velocity $v = v_0$ at time $t = 0$, and there are no external forces, on what time scale does this initial velocity decay? Once the particle comes to equilibrium with its surroundings, what is its typical speed? For simplicity, assume that the motion is in one dimension, and neglect the force of gravity on the particle.

b) If you observe this system under a microscope with a conventional video camera, you can measure the position of the particle 30 times per second, and if you work hard at the image processing you can obtain a positional accuracy of $\sim 1\ \mu$m. If you estimate the velocity by taking differences of position from frame to frame, will you see the velocity that you computed in (b)? Why or why not? Would it be worth buying a camera that captures 500 frames per second?

c) Give a quantitative description of the particle's position vs. time that you expect to see in such an experiment. When you sketch the expected results, be sure to indicate (at least approximately) the scales on the axes. Would buying a camera that captures 500 frames per second reveal any new features of this trajectory?