M08T.1 - Brownian Motion

Problem

A spherical object of radius a and mass M is undergoing Brownian motion at temperature T in a fluid of viscosity η. The fluid and the object have identical density, so there is no buoyancy force. When a is large compared to the size of the molecules in the fluid, this motion is given by the Langevin equation of motion for the velocity $\vec{v}(t)$ of the object:

$$M\frac{d\vec{v}(t)}{dt} = \vec{F}(t) - 6\pi\eta a\vec{v}(t)\,,$$

where $\vec{F}(t)$ is the instantaneously random force on the object at time t due to collisions with the molecules of the fluid.

a) Solving this Langevin equation (and also using equipartition of energy), calculate the autocorrelation function $\langle\vec{v}(0)\cdot\vec{v}(t)\rangle$ of the object's velocity.

b) Use your result from (a) to calculate the leading long-time behavior of the mean-square displacement of this object after time t, namely $\langle(\vec{r}(t)-\vec{r}(0))^2\rangle$, and show how measuring this quantity permits an experimental determination of Boltzmann's constant k_B.