M08M.3 - Gravitational Capture of Dark Matter Particles

Problem

a) A particle of mass m is in a circular orbit around the sun at the radius of the Earth's orbit, traveling at a speed $v_\oplus = 29.8$ km/s. Neglecting the Earth's gravitational potential, compute the escape speed $v_{\rm esc}^\odot$ from the solar system at the radius of the orbit. Compute the escape velocity in the frame of the Sun.

b) What is the minimum speed $v_{\rm min}$ of a particle just escaping the solar system computed in a frame corotating with the particle's original circular orbit. Here, the corotating frame is the frame moving with the orbital velocity of the Earth. Neglect Earth's gravitational potential.

c) Now assume that the particle is at rest in the bottom of the potential well of the Earth. If the escape speed from the Earth at the Earth's surface is $v_{\rm esc}^\oplus(R_\oplus = 11.2$ km/s, what is the escape speed $v_{\rm esc}^\oplus(0)$ from the Earth for a particle at its center, computed in the Earth's frame? Assume the Earth is a sphere of uniform density. Here the Earth's radius is denoted $R_\oplus$.

d) A particle of mass m has a velocity v relative to the Earth as it traverses the solar system at the orbital radius of the Earth around the Sun. The initial velocity v is the value far enough outside the gravitational well of Earth that the Earth's gravitational effects need to be accounted for in what follows.

The particle takes a trajectory that passes through the center of the Earth, and at the very center elastically scatters off of an iron atom, having a mass $m_{\rm Fe} = 52$ GeV/c^2. What is the maximum particle mass m that a particle can have and still be able to be gravitationally captured by the Earth subsequent to elastically scattering off of an iron atom at the center of the Earth? To compute m, assume the particle has the minimum velocity v such that it is not bound to the solar system.