J09E.1 - Motion in EM Fields

Problem

In a large region of space there is a uniform magnetic field B in the z-direction and a uniform electric field E in the x-direction. A particle of mass m and charge q is initially at rest at the origin.
The equation of motion is

$$m\frac{dU^\alpha}{d\tau} = qF^\alpha{}_\beta U^\beta$$

where τ is the proper time of the particle and $U^\alpha = dx^\alpha/d\tau$ is its four-velocity. The field strength tensor is $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$, where A^α is the 4-vector potential (its time component A^0 is the electric potential ϕ). Note that in this problem we use units where the speed of light $c = 1$.

a) Solve for U^μ as a function of the proper time of the particle assuming that $B^2 > E^2$. What is the average 4-velocity of the particle?

b) Solve for the particle position x^μ as a function of the proper time.