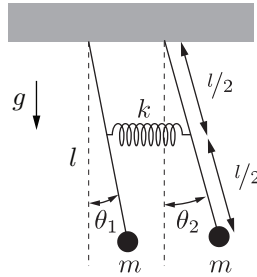


## J09M.1 - Coupled Pendula

### Problem

Two simple pendula, each of length  $l$  and mass  $m$ , are coupled by a spring of force constant  $k$ . The spring is attached to the rods of the pendula, which are massless and inextensible, at their half-way points, as shown. Throughout, assume the angles  $\theta_1$  and  $\theta_2$  are small and that motion is confined to the 2D plane.



- What are the normal frequencies of the system, and the corresponding normal mode vectors?
- Consider now the case of “weak coupling”—*i.e.*, the case when  $k$  is small. With respect to what is  $k$  small?

At  $t = 0$  the lefthand pendulum is displaced by an angle  $\theta_1(0) = \theta_0$  and released from rest; the righthand pendulum is at rest with  $\theta_2(0) = 0$ . Find expressions for  $\theta_1(t)$  and  $\theta_2(t)$  for  $t > 0$ . How long will it take before the lefthand mass stop swinging and the righthand mass achieves maximum amplitude?