J09M.2 - Minimizing Drag

Problem

The goal of this problem is to determine the optimum shape of a body in order to minimize the drag from a constant flow of air. Suppose that the body has cylindrical symmetry (that is, it is invariant under rotations around the z axis), and has a height L. If its radius is given by $r(z)$, a good approximation to the drag is the expression

$$D = a \int_0^L r(z) \left(\frac{dr}{dz} \right)^3 dz \,,$$

where $a > 0$ is some constant.

a) If $r(0) = 0$ and $r(L) = d$, what is the optimal shape of the body in order to minimize D?

b) Suppose, in addition, that the body has a fixed volume V. How would you find the optimal shape under this constraint? Find a first order differential equation that the optimizing $r(z)$ will satisfy.