J09Q.2 - Scattering an Electron from a "Step" in the magnetic field

Problem

For $x < 0$, the magnetic field is $\vec{B} = 0$; for $x \geq 0, \vec{B} = B_0\hat{z}$ is uniform and pointed along the z axis. An electron with its spin oriented along $\hat{z}$ is incident from $x < 0$ with velocity $v\,\hat{x}$ and scatters from the field. Be sure to include both the interaction of the electron's charge and its magnetic moment with the field.

a) What is the scattering wavefunction in the semi-classical WKB approximation in the classically-allowed region? Although you may leave the overall amplitude unnormalized, be sure to get the relative amplitude and phase of the incident and scattered waves correct. Clearly **state what gauge you use**, and **state what quantity the incident speed v must be much larger than** for the semi-classical WKB approximation to be appropriate in the region $x \geq 0$. Your expression for the wavefunction may involve an integral that can be performed, but carrying out the integral results in little simplification of the formula, so first give your result with the integral not performed and only carry out the integral if you have plenty of time.

b) Explicitly show that for this wavefunction all components of the velocity of the electron at each position agree with the classical trajectory.