J09E.3 - Magnetic Pressure

Problem

Consider an infinite cylinder of conducting material with an axial current density $J_z = J(r)$ and a resulting azimuthal magnetic field $B_\phi = B(r)$. The coordinate r is the radial coordinate with respect to the axis of the cylinder. Assume the current is confined within the finite radius R of the cylinder. The integral of the current density over the cross-sectional area of the cylinder is the total current I.

a) Use Ampère's law to determine $B_\phi = B(r)$ for $r \geq R$ in terms of I and r.

b) Assume the current density is constant within the cylinder. Find $B(r)$ for $r < R$.

c) The Lorentz forces on the current will "pinch" the material of the conductor and try to squeeze it radially inward. These pinching forces are balanced by non-magnetic pressure gradient forces (for example, elastic forces for a solid metal, or compressed-gas pressures for a plasma). Find an expression for the pressure, $p = p(r)$, inside the conductor for the uniform current distribution of part b), and sketch the dependence of p on r. You can assume that the pressure is zero at the surface of the conductor.

d) Calculate and sketch the pressure for the case that the current flows in a very thin, uniform layer along the surface of the cylinder.