J09Q.3 - Trapping Atoms in an Optical Lattice

Problem

A 1-D optical lattice uses counter-propagating light beams to produce a standing wave, which has a periodic intensity pattern. Through the AC Stark-shift effect, the system energy is a function of the positions of the atoms. To simplify the problem, let's consider a hypothetical two-level atom with a ground state $|1\rangle$ and an excited state $|2\rangle$. The energy difference between the two states is $\hbar\omega_0$. Now an optical field $E(x,t) = E_0 \cos kx \cos \omega t$ is introduced, and most of the atoms are in state $|1\rangle$ right before the optical field is turned on. The frequency ω is close to ω_0. The optical field interacts with atoms through the electric dipole coupling $\langle 2|E \cdot D|1\rangle = E \cdot \langle D\rangle$. Here, D is the dipole operator, and $|E \cdot \langle D\rangle| \ll |\hbar\omega_0 - \hbar\omega|$.

a) Please find the perturbed energy of state $|1\rangle$ due to the optical field as a function of position. How deep is the potential well produced by the optical lattice?

b) Where do the atoms tend to be trapped at a different optical frequency ω?