M09Q.2 - Dynamics of Spin-1/2 Particles (J94Q.2)

Problem

The spin dynamics of a spin-1/2 particle is governed by the Hamiltonian

$$H = \lambda\hbar\sigma_x\,.$$

At the initial time $t = 0$ the spin is pointing up along the z axis. We are going to be concerned with measurements of the z-component of the spin at later times. Recall that in the standard representation

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\,.$$

We want to compare two situations:

a) The z-component is measured at time τ. What is the probability — call it P_I — that the spin will be pointing down?

b) The z-component of spin is measured at an intermediate time $\tau/2$. This measurement is carried out by a compatriot who assures you that it was done but who doesn't tell you his finding. You carry out your own measurement at time τ, i.e., at a time $\tau/2$ after the intermediate measurement. What is the probability — call it P_{II} — that the spin is pointing down?