M09Q.3 - Pion-Atom Scattering (J94Q.3)

Problem

Consider the following 1-dimensional version of inelastic scattering of a projectile off an atom. In the 1-d case one speaks of transmission-reflection coefficients rather than differential scattering cross sections.

Our 1-d atom consists of an "electron" of mass m, position variable x_e, moving in a "nuclear" potential $V(x_e)$. Let $u_0(x_e)$ and ϵ_0 be the (normalized) ground state eigenfunction and energy; let $u_1(x_e)$ and ϵ_1 be the eigenfunction and energy of the first excited bound state. The projectile — a "pion" — has mass M, position variable x_p, and incidente energy $E = \hbar^2 k^2/2M$. The projectile interacts with the electron with potential $W(x_e - x_p$; the projectile does no interact with the nucleus.

Consider the process in which the incident pion collides with the atom, which is initially in its ground state. The projectile is reflected in the backward direction with energy $E' = \hbar^2 k'^2/2M$ and leaves the atom in the first excited state. You are given the energies of the atomic levels; so, given k, you know the momentum, $-\hbar k'$, of the reflected projectile. In addition to k and k' you are *given* the following (possibly complex) functions:

$$\tilde{V}(q) = \int dx\, e^{iqx} V(x), \quad \tilde{W}(q) = \int dx\, e^{iqx} W(x), \quad f(q) = \int dx\, u_1(x) e^{iqx} u_0(x)\,.$$

For any of these functions you use, you must specify the argument q in terms of the given quantities, e.g., k and k'.

Compute the reflection coefficient R for this process, in the lowest Born approximation.

Remark: The reflection coefficient is simply the transition rate to the final state under consideration, divided by the incident projectile flux.