J10Q.1 - Harmonic Oscillator

Problem

Consider an isotropic three-dimensional harmonic oscillator described by the rotationally-invariant Hamiltonian

$$H = \frac{|\vec{p}|^2}{2m} + \frac{m\omega^2}{2}|\vec{v}|^2 \,.$$

a)

i. What are the energies and degeneracies of the lowest three energy levels?

ii. Account for the degeneracies by classifying states in these levels into total angular momentum multiplets.

b) By how much does the ground state energy change under the influence of a perturbation of the form

$$H' = \lambda(\vec{b} \cdot \vec{x})^3$$

where $\vec{b}$ is some fixed vector, and λ is small? Calculate the correction up to second order in λ.

Now suppose that the oscillating particle has charge q. At time $t = 0$, a weak uniform electric field $\vec{E}$ is switched on, which then slowly decays as $\vec{E}(t) = \vec{E}_0 e^{-t/\tau}$, with $\tau > 0$.

c) What is the probability (to leading order in $|\vec{E}_0|$) that a system originally in the ground state will be in an excited state at a much larger time $t \gg \tau$?