# J10T.1 - Graphene

## Problem

Graphene is a two-dimensional sheet of carbon atoms. Both electronic and phonon degrees of freedom contribute to the low-temperature specific heat per unit area. The electron states resembe the states of the massless Dirac equation, with energies

$$\varepsilon_{\pm}(\vec{p}) = \varepsilon_0 \pm v_{\mathrm{F}} p\,, \quad p \equiv |\vec{p}|\,,$$

where $\vec{P} = (p_x, p_y)$ is the analog of the momentum carried by a Dirac electron. (There are two energy bands, $\varepsilon_+(\vec{p}) \geq \epsilon_0$ and $\varepsilon_-(\vec{p}) \leq \epsilon_0$ which become degenerate at $\vec{p} = 0$). These states have a fourfold degeneracy (the usual two-fold spin degeneracy is doubled by an additional "valley" index).

a) If the Fermi energy $E_{\mathrm{F}}$ is $\varepsilon_0 + v_{\mathrm{F}} p_{\mathrm{F}}$, with $p_{\mathrm{F}} > 0$, what is the leading behavior of the electronic specific heat as $T \to 0$?

b) What is the low-temperature electronic specific heat when $p_{\mathrm{F}} = 0$?

(The next calculation is independent of parts a), b) above).

Recently, freely suspended graphene sheets have been studied. These have an unusual phonon spectrum: in addition to longitudinal and transverse sound waves with frequencies $\omega = v_{\mathrm{L}} q, v_{\mathrm{T}} q$ (where $q$ is the magnitude of the wavenumber $\vec{q}$), there is an extra low-frequency mode $\omega = Kq^2$ where atomic displacements are normal to the sheet.

c) Obtain the leading behavior of the phonon contribution to be specific heat as $T \to 0$.

You may express your answers in terms of the numerical constants

$$C_n^{\pm} = \int_0^{\infty} dx \frac{x^n}{e^x \pm 1}\,, \quad n > 0\,.$$