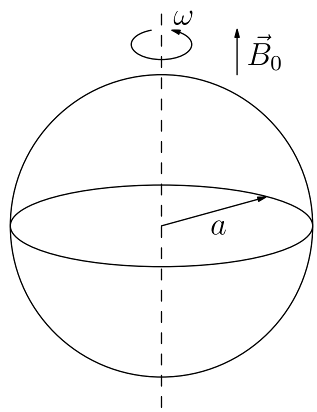
J10E.2 - Rotating Sphere in a Magnetic Field

Problem



A solid metallic sphere of radius a has finite conductivity, carries no net electric charge, and is free to rotate without friction about a vertical axis through its center. The region outside the sphere is vacuum. There is a uniform magnetic field with flux density $\vec{B}_0$ parallel to the axis.

The sphere is given an impulse that starts it spinning around the axis and there is some initial Ohmic dissipation. After the dissipation has ceased, the sphere is in a steady state of rigid rotation with constant angular velocity ω_∞.

In steady state, *to lowest order in both B_0 and ω_∞*, find:

a) The electric field $\vec{E}(\vec{r})$ and electric potential $\Phi(\vec{r})$ in the *interior of the sphere*, $r < a$. (Give these in the non-rotating "laboratory frame".)

b) The electric potential *outside* the sphere. (Express your answer in spherical coordinates (r, θ, ϕ).) State the nature of the electric field it describes (*i.e.*, monoole, dipole, quadrupole, *etc.*).

c) The induced bulk and surface charge density distributions in the conductor that give rise to this potential.

Note: By working to lowest order in B_0 and ω_∞, you can ignore both the mechanical deformation of the metal sphere due to rotation and the magnetic fields generated by currents in the metal (therse are negligibly small relative to B_0).